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Term Structure of Interest Rates Review of a Theory of the Term Structure of Interest Rates (CIR)

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Abstract

Term structure of interest rates is a calculation of the relationship between the yields on securities which only differ in their term to maturity. This relationship has several determinants among them interest rates and yield curve. Economists and investors believe that the shape of the yield curve reflects the market's future expectation for interest rates and the conditions for monetary policy. This paper reviews the work of Cox, Ingersoll and Ross of 1985 in their handling of term structure of interest rates as well as look at different models of term structure of interest rates both single factor and multifactor models.

Keywords: **Interest Rates, Economists, Investors**

INTRODUCTION

The term structure of interest rates measures the relationship among yields on securities that differ only in their term to maturity. The determinants of this relationship have long been a topic of concern to economists. By offering a complete schedule of interest rates across time, the term structure embodies the market's anticipations of future events for hedging purposes and policy implications. An explanation of the term structure gives a way to extract this information and to predict how changes in the underlying variables will affect the yield curve. Thus, we can say that the term structure of interest rates refers to the relationship between bonds of different terms. A number of stylized facts about interest rate volatility have been uncovered in literature. First, interest rate volatility is clearly stochastic. Second, interest rate volatility contains important unspanned components Benzoni [1]. Third, changes in interest rate volatility are correlated with changes in interest rates. For instance, estimates in Andersen and Lund [2] and Ball and Torous [3], who both study the dynamics of the short-term interest rate, imply that relative interest rate volatility is negatively correlated with interest rates while absolute interest rate volatility is positively correlated with interest rates. Fourth, the unconditional (realized and implied) volatility term structure exhibits a hump [4].

Here, yield curve is constructed by plotting the interest rates of bonds against their terms. For instance, term structure can be defined as the yield curve which is displaying the relationship between spot rates of zero coupon securities and their term to maturity. As can be seen, there is a strong connection between interest rates and yield curve. We can ask ourselves what makes the term structure of interest rates important. Economists and investors believe that the shape of the yield curve reflects the market's future expectation for interest rates and the conditions for monetary policy. Modeling of term structure of interest rates helps in assigning economic interpretations of the interest rate behavior approximated with affine models in terms of monetary and real economic factors.

One Factor Models

One factor models were the first step in modeling the term structure of interest rates. These models are grounded on the estimation of bond yields as functions of the short term interest rate. Two of the most popular bond pricing models are those constructed by Vasicek and Cox, et al. [5]. Each of these models has a single factor with bond price depending on a single variable, spot interest rate, r.

Vasicek's Model

In discrete time, the single state variable z follows a first order auto regression

$Z_{t+1} = \zeta Z_t + (1-\zeta)\theta + \sigma \varepsilon_{t+1}$

$= Z_t + (1-\zeta)(\theta - Z_t) + \sigma \varepsilon_{t+1}$ with $\{\varepsilon_{t+1}\}$ distributed normally and independently with mean zero and variance one. The mean of Z is θ. The conditional variance is σ^2 and the unconditional variance $\sigma^2/(1-\zeta^2)$. The parameter ζ controls mean reversion: If ζ =1, Z is a random walk and shows no tendency to return to any specific value. But if $0 < \zeta < 1$, Z is expected to return to its mean value of θ at rate 1- ζ.

Cox- Ingersoll- Ross Model

The CIR model has a similar structure. The difference lies in the behavior of the state variable Z: In the Vasicek model the conditional variance is constant, while in the CIR it varies with the state. The unconditional mean of Z is θ. The conditional variance is;

$Var_t(Z_{t+1}) = Z_t \sigma^2$, which has a mean of $\theta\sigma^2$. The unconditional variance is $Var(Z) = \theta\sigma^2/(1-\zeta^2)$.

These models have four parameters: three governing the dynamic behavior of the state variable and one controlling the market's valuation of risk. These one factor models are a good place to start but not a good place to stop. There are too many discrepancies between them. One discrepancy in the Vasicek and CIR models is the shape of the mean yield curve: If ζ is chosen to reproduce the autocorrelation of the short rate, the mean yield curve is substantially less concave in the models than it is in the data. This discrepancy was pointed out by Gibbons.

Another discrepancy is the pattern of autocorrelations. Both of these models are linear: yields of all maturities, yield spreads, and, indeed, all linear combinations of yields are linear functions of Z. As a consequence, they share with z its autocorrelations than the short rate. Further to this, yet another discrepancy is that innovations in Z are conditionally normal. Evidence suggests to the contrary that interest rate innovations have substantial excess kurtosis. However, one factor models do not overcome the discrepancy between the theoretical mean yield curve implied by the time series properties of bond yields and the observed curves that are substantially more concave than implied by theory. The solution to this discrepancy is the multifactor affine models with stochastic volatility [6].

Multifactor Models

Schwartz [7] develops a flexible stochastic multifactor model of term structure of interest rates with 3 factors featuring multiple unspanned stochastic volatility of 3 factors and non-zero correlation between innovations to forward rates and their volatilities. The model is estimated by quasi-maximum likelihood in conjunction with the extended Kalman filter on an extensive panel dataset of LIBOR and swap rates, ATMF swaptions, ATMF caps, and non-ATMF caps with a very good fit to the data. With the use of the flexible extended affine market price of risk specification, Schwartz obtains a tractable description of the dynamics of the term structure under the actual measure making the model useful in risk management applications involving portfolios of interest rate derivatives. This model could as well be used in valuation of mortgage backed securities due to its careful modeling of stochastic volatility, which is a key determinant of the value of the prepayment option.

These three-factor models suffer from admissibility drawbacks because factors determining volatility must be positive. The estimation of volatility parameters is very difficult to estimate leading to different term structures. Buhler et al. [8] in their analysis of German term structure through principal component analysis reveal that two factors explain more than 95% of the variation in the German term structure of interest rates.

STATEMENT OF THE PROBLEM

In a world of certainty, equilibrium forward rates must coincide with future spot rates, but when uncertainty about future rates is introduced the analysis becomes complex. Previous theories of term structure have taken the certainty model as their starting point and have proceeded by examining generalizations of the certainty equilibrium relationships. CIR considers the problem of determining term structure as being a problem in general equilibrium theory. Anticipation of future events is important, as are risk preferences. Individuals could have specific preferences about the timing of their consumption, and thus have a preferred habitat. Their model thus permits detailed descriptions about how changes in a wide range of underlying variables will affect the term structure.

RESEARCH OBJECTIVES

1. To investigate the determinants of term premiums.
2. To analyze statistically ex ante propositions by using ex post data

RESEARCH QUESTIONS

1. What determines term premiums
2. What is the relationship between ex ante propositions and ex post data

THEORETICAL REVIEW OF TERM STRUCTURE

There are four well-known theories of term structure (Figure 1):

a. Expectations Theory,
b. Liquidity Preference Theory,
c. Market Segmentation Theory
d. Preferred Habitat Theory.

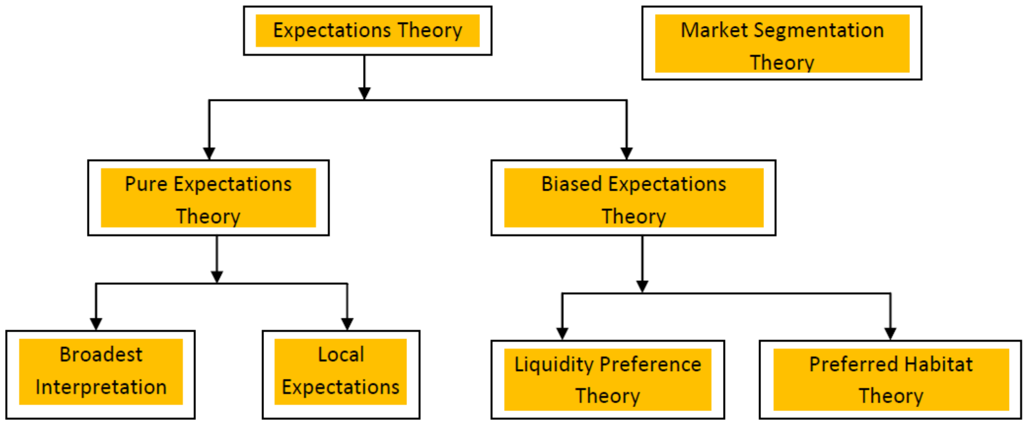

Figure 1: Theoretical review of term structure

EXPECTATIONS THEORY (PURE EXPECTATIONS THEORY)

The theory asserts that a long term rates constitute an average (a weighted average in the case of coupon bearing securities) of expected future short term rates. It says that forward rates (or marginal rate of interest) constitute unbiased estimates of future spot rates. Investor's expectations of future interest rates alone create the shape of the yield curve.

It implies that the expected value of the returns derived from holding long and short term securities for identical time periods are the same. The key assumption behind this theory is that buyers of bonds do not prefer bonds of one maturity over another, so they will not hold any quantity of a bond if its expected return is less than that of another bond with a different maturity. Bonds that have this characteristic are said to be perfect substitutes.

Note that what makes long term bonds different from short term bonds are inflation and interest rate risk. Therefore this theory assumes away inflation and interest rate risks. According to this theory, long term rates are all averages of expected future short term rates. For example,

Suppose one year rates over the next five years are 5%, 6%, 7%, 8% and 9%. Then

interest rate on a two year long bond is 5% + 6%/2 = 5.5%
And, interest rate on five year long bond is 5% + 6% +7% + 8% +9%/2
= 7%

According to this theory, an average smoothens out large volatilities and therefore, if the current short term rate changes it will have very little impact on a long term rate. Thus, short term rates are more volatile than long term rates. But expectations theory cannot explain why long term yields are normally higher than short term yields, in other words, why the yield curve is usually upward sloping. If the short rates are low now, they are expected to go up in future.

The **broadest interpretation** is that given any investment horizon, investors expect the same return, regardless of the maturity of the investment vehicle selected. This ignores the price risk associated with selling a bond prior to its maturity.

The **local interpretation** suggests that the return on bonds with different maturities will be identical over a short-term investment period, commencing immediately. This is the only interpretation of the pure expectation theory that can be sustained in equilibrium.

According to Backus et al. 1997, expectations theory holds if in the following forward regression the slope c_n=1, $f_{n-1, t+1} - y_{1,t}$ = constant + $c_n (f_{n,t} - y_{1,t})$ + residual. This is because the expectations theory of term structure holds with constant term premiums in the form of: $f_{n,t} = E_t (y1,t +n) + \Lambda n$

Liquidity Preference (Premium) Theory by Hicks [9]

This theory is one of the two forms of biased expectations theory. Duration measures the price risk of holding a bond. Duration increases as the bond's maturity lengthens. Risk aversion will cause forward rates to be systematically greater than expected spot rates, usually by an amount increasing with maturity. The term premium is the increment required to induce investors to hold longer term securities. Even default free bonds are risky because of uncertainty about inflation and future interest rates. Bond holders care about purchasing power of the return- real return-not just the nominal value of the coupon payments. Uncertainty about inflation creates uncertainty about a bond's real return, making the bond a risky investment.

The further we look into the future, the greater the uncertainty about the level of inflation, implying that a bond's inflation risk increases with its time to maturity. Interest rate risk arises from a mismatch between investor's investment horizon and a bond's time to maturity. If a bond holder plans to sell a bond prior to maturity, changes in the interest rate generate capital gains or losses.

The longer the term of the bond, the greater the price changes for a given change in interest rates and the larger the potential for capital losses. As in the case of inflation, the risk increases with the term to maturity, so the compensation must increase with it. The buyer of long term bonds would require compensation for the risks they are taking buying long term bonds. Like the expectations theory, this theory predicts that interest rates of different maturities will move together because the long term rates are essentially tied to the short term rates.

And finally, since the risk premium increases with time to maturity, the liquidity premium theory tells us that the yield curve will normally slope upwards, only rarely will it lie flat or slope downwards. If we consider this theory in a different way, we can say that the longer-term interest rates not only reflect investors' future assumptions for the interest rates, but also include a premium for holding these longer-term bonds which we state as term premium or liquidity premium.

Market Segmentation Theory (Segmented Markets Theory) by Culbertson [10]

This theory assumes that markets for different maturity bonds are completely segmented. The interest rate for each bond with a different maturity is then determined by the supply of and demand for the bond with no effects from the expected returns on other bonds with other maturities. Individuals have stronger maturity preferences and bonds of different maturities trade in separate and distinct markets. Longer bonds that have associated with them inflation and interest risks are completely different assets than short term bonds. Thus the bonds of different maturities are not substitutes at all, so the expected returns from a bond of one maturity has no effect on the demand for a bond of another maturity. Because bonds of shorter holding periods have lower inflation and interest rate risks, segmented market theory predicts that yield on longer bonds will generally be higher, which explains why the yield curve is usually upward sloping.

However, since markets for different maturity bonds are completely segmented, there is not a reason why the short and long yields should move together. This theory, just like the preferred habitat theory, agrees that lenders and borrowers have preferred maturity ranges and there is no premium large enough to induce investors out of their preferred maturity range.

Preferred Habitat Theory by Modigliani and Sutch (1966)

This theory is the other of the two forms of the biased expectations theory. Preferred habitat theory is the combination of the market segmentation theory and expectations theory, because investors care about both expected returns and maturity of their securities. Additionally, because investors have different investment horizons and buy bonds with maturities outside their habitat, they need a meaningful premium. Thus, this theory allows market participants to trade outside of their preferred maturity if adequately compensated for the additional risk.

But, we have to remember that investors prefer short-term to long-term bonds and never prefer a long term bond if this offers the same expected return as a series of short-term bond. Here, short-term investors are more prevalent in the fixed-income market, thus longer-term rates tend to be higher than short-term rates.

HISTORICAL DEVELOPMENTS

MERTON 1973

Capital asset pricing model is the one of the most important developments in capital market theory. It is known as the Sharpe- Lintner-Mossin mean-variance equilibrium model of exchange. Even though this model is considered among many papers, it is criticized too much.

However, the model is still being used because it is an equilibrium model which provides a strong specification of the relationship among asset yields that is easily interpreted, and the empirical evidence suggests that it does explain a significant fraction of the variation in asset returns.

Merton developed an equilibrium model of the capital market which is an inter-temporal consumer-investor behavior based model. This model provides a specification of the relationship among yields that is more consistent with empirical evidence.

Vasicek, 1977

Vasicek gives an explicit characterization of the term structure of interest rates in an efficient market. The model is widely used for pricing the bonds. This model is a one-factor model which means that rates depend on the spot interest rate. Thus, the spot rate defines the whole term structure. This model has the advantage that it can be used to value all interest-rate-contingent claims in a consistent way. Its main disadvantage is that it involves several unobservable parameters and do not provide a perfect fit to the initial term structure of interest rates.

Cox, Ingersoll, Ross (CIR) [5]

The researchers developed an inter-temporal general equilibrium asset pricing model. We know that the effective concepts when determining the bond prices are risk aversion, investment alternatives, anticipations and preferences about the timing of consumption. The researchers considered the problem of determining the term structure as being a problem in general equilibrium theory, and their approach contains elements of all of the previous theories. Anticipations of future events are important, as are risk preferences and the characteristics of other investment alternatives. Also, individuals can have specific preferences about the timing of their consumption, and thus have, in that sense, a preferred habitat.

Thus, their model permits detailed predictions about how changes in a wide range of underlying variables will affect the term structure. This model has the same main advantage and disadvantage as Vasicek's model.

Ho, Lee [11]

They propose an alternative approach to pricing models through a binomial model. The approach is taking the term structure as given, and deriving the feasible subsequent term structure movements. These movements must satisfy certain constraints to ensure that they are consistent with an equilibrium framework. Specifically, the movements cannot permit arbitrage profit opportunities. They called these interest rate movements arbitrage-free rate movements. When the arbitrage rate movements are determined, the interest rate contingent claims are then priced by the arbitrage methodology. Therefore, their model is a relative pricing model in the sense that they price the contingent claims relative to the observed term structure; however, they do not endogenize the term structure. Thus, Ho and Lee pioneered a new approach by showing how an interest rate model can be designed so that it is automatically consistent with any specified initial term structure.

Hull, White (extended Vasicek) 1990, Hull, White (extended CIR), 1990

The researchers showed that the one-state-variable interest-rate models of Vasicek and Cox, et al. [5] can be extended so that they are consistent with both the current term structure of interest rates and either the current volatilities of all spot interest rates or the current volatilities of all forward interest rates. The extended Vasicek model is shown to be very tractable analytically. The article compares option prices obtained using the extended Vasicek model with those obtained using a number of other models. Besides, the researchers present two one-state variable models of the short-term interest rate. Both are consistent with both the current term structure of interest rates and the current volatilities of all interest rates. In addition, the volatility of the short-term interest rate can be a function of time.

The user of the models can specify either the current volatilities of spot interest rates (which will be referred to as the term structure of spot rate volatilities) or the current volatilities of forward interest rates (which will be referred to as the term structure of forward rate volatilities). The first model is an extension of Vasicek. The second model is an extension of Cox.

Black, Karasinski [12]

Black et al. describe a one-factor model for bond and option pricing that is based on the short-term interest rate and that allows the target rate, mean reversion and local volatility to vary deterministically through time. For any horizon, the distribution of possible short rates is lognormal, so the rate neither falls below zero nor reflects off a barrier at zero. A model like this allows one to match the yield curve, the volatility curve and the cap curve. Surprisingly, adding to future local volatility lowers the volatility curve.

A conventional binary tree with probabilities of 0.5 but variable time spacing is used to value bonds and options. When the inputs are constant, the slope of the yield curve starts out positive and ends up negative, while its curvature shifts from negative to positive. Even when mean reversion is zero, the volatility curve has a negative slope. The researchers presented a one-factor model of bond prices, bond yields, and related options. The single factor that is the source of all uncertainty is the short-term interest rate. They assumed no taxes or transaction costs, no default risk and no extra costs for borrowing bonds.

They also assumed that all security prices are perfectly correlated in continuous time.

Here, before moving to the Health-Jarrow-Morton's approach would like to consider what we learned so far. Generally, the term structure models which are prior to Health-Jarrow and Morton were finite dimensional Markovian models. In Markovian models, the interest rate economy is determined by the spot rate and besides, but not necessarily, one or two additional state variables.

This enabled the use of standard arbitrage arguments, along the lines of Black and Scholes and Merton to derive the PDE for the bond and bond option prices which, in turn, enabled the application of well-developed techniques from the theory of PDEs to obtain analytic solutions, and numerical solutions in cases where this was not possible. The progenitors of this approach could be regarded as Vasicek and Brennan. After Vasicek's model is established, many of the interest rate models are proposed using this model. In Vasicek's model, the spot rate was assumed as a mean reverting process with constant volatility and constant mean reversion level. As I stated before, the common tool used in these models was the no-arbitrage arguments of Black-Scholes and Merton, which produced the pricing partial differential equation for the bond, and bond option, prices in a systematic manner. Well-developed techniques from the theory of partial differential equations were then applied to solve, either analytically or numerically, these pricing equations.

These early models are useful because they have analytic solutions.

However, the calibration of model parameters to observed market data is a non-trivial task. Especially, many models cannot be calibrated consistently to the initial yield curve. Additionally, the relationship between the model parameters and the market observed variables are not always clear and we cannot always incorporate observer market features, such as humped volatility curve, into these models.

The quantity driving this class of models was the instantaneous spot rate of interest, and, since the spot rate is a non-traded quantity, these models usually involved the market price of interest rate risk.

Heath, Jarrow, Morton, HJM [13]

By contrast, the Heath, Jarrow and Morton model provides us a very general interest rate framework which is capable of incorporating most of the market observed features. This model takes as the quantities driving the model the continuum of instantaneous forward rates, which are directly related to the prices of traded bonds. Furthermore, the

HJM models are automatically calibrated to the initial yield curve, and the connection between the model parameters and the market variables often emerge from the theory. They used techniques from stochastic calculus to construct a very general framework for the evolution of interest rates that had the useful feature that the model is naturally calibrated to the currently observed yield curve. Although the HJM model is widely accepted as the most general and consistent framework under which to study interest rate derivatives, the added complexity and the absence of efficient numerical techniques under the general HJM framework saw the earlier models retain their popularity, particularly among practitioners. The main drawback of the HJM model is that these models are non-Markovian in general, and as a result, the techniques from the theory of PDEs are no longer applicable to these models.

MAJOR EMPIRICAL STUDIES

Macaulay

He was among the first to produce empirical evidence that related long term rates to expectations of future short term rates. He found that time money rates did in fact anticipate the seasonal rise in call money rates and concluded that this constituted evidence of definite and relatively successful forecasting. He was nevertheless unable to uncover additional evidence of successful forecasting saying that successful forecasting is rare because successful forecasting is also rare.

Hickman

In his unpublished but widely cited and read manuscript of 1942, he sought evidence of successful forecasting by comparing observed or actual yield curves with those predicted one year or more ahead by the term structure of interest rates. He found that simply assuming that this year’s yield curve will be the same as next year’s gave what he regarded as better predictions of subsequently observed yield curves than the expectations hypothesis. This was one of the earlier uses of inertia hypothesis as a benchmark for evaluating the predictive content of a substantive hypothesis.

Culbertson

His empirical research is similar to Hickman's; both ran tests based on the assumption that forward rates are accurate predictions of future spot rates. He examined yields of short and long term government securities for identical periods of time. He argued that if the expectations hypothesis is valid, then yields to investors ought to be the same whether short or long term securities are held. He found marked differences in returns for the same holding periods. Since he found it difficult to believe that speculators would operate in the government securities markets and predict as badly as his results suggested, he rejected the expectations hypothesis.

Walker

His work deals with governmental interest rate policy during world war two when the FED and Treasury embarked upon a policy of stabilizing, through open market operations and the maturity composition of new issues, the existing levels of rates on government securities. At that time, the yield curve was sharply rising. If the expectations hypothesis is correct, the prestabilization term structure implied that future short term rates were expected to be higher than existing short term rates.

In contrast, the stabilization policy implied that future short term rates would be the same as current short term rates. When the financial community became convinced that existing long term rates were inconsistent with revised expectations of future short term rates: long term rates were too high. Hence, there was a tremendous shift out of short into long term securities by the holders of governmental obligations. Walker's results unlike Macaulay's findings, cannot be interpreted as providing unambiguous support for the expectations hypothesis because they are also consistent with an implication of the liquidity preference hypothesis. Although Walker's results do not discriminate between expectations and liquidity preference, they do discriminate between expectations and liquidity preference on the one hand and market segmentation on the other. If the holdings of governments by the major institutions of the financial community changed as much as walker reports they did, this constitutes evidence against the market segmentation hypothesis; if the market segmentation hypothesis is correct, Walker should not have observed a shift in the maturity distribution of governments by the major institutions of the financial community.

MEISELMAN

He is the first investigator to employ an operational test of the expectations hypothesis that does not depend upon accurate foresight for its validity. If a relationship exists between expectations and the term structure of interest rates, then its existence can be detected despite inaccurate predictions showed that expectations, whether or not they are correct, nevertheless affect the term structure of interest rates.

EMPIRICAL EVIDENCE

Shea investigated the long memory of interest rates in the context of the expectations hypothesis of the term structure. He found that allowing for the possibility of long memory significantly improves performance. Backus [14] observed that the volatility of bond yields does not decline exponentially when the maturity of the bond increases.

Lai and Philips provide evidence based on semi parametric methods that ex-ante and ex-post US real interest rates are fractionally integrated.

Nandwa examined whether normal interest rates in a sample of sub-Saharan countries follow stochastic trends or unit root processes and whether the Fisher hypothesis holds in the area. Aboagye et al. [15] investigated the question of optimal spread between bank lending rates and deposit rates in Ghana. They found that increases in market power, bank size, staff costs among other factors significantly increase net interest margins, while increases in bank excess cash reserves and central bank lending rate decrease these factors.

Elliot [16] estimated an econometric model including interest rates for Kenya. Musila applied cointegration methods to develop a macro model for forecasting purposes. Ndungu examined the relationship between exchange rates and interest rate differentials in Kenya using a time varying parameters approach.

Odhiambo investigated the impact of interest rate reforms on financial deepening and economic growth in Kenya. He found a positive relationship in both cases using standard (1(0)/1(1) cointegration techniques.

RESEARCH METHODOLOGY

As a rational asset pricing model used to study term structure of interest rates, this model uses arbitrage method of bond pricing developed by Vasicek, which considers the variables on which price depends, properties of endogenously determined underlying variables and the exact form of the factor risk premiums.

In this model the prices of bonds of all maturities depend on a single random explanatory factor, the spot interest rate. As a single factor model, price changes in bonds of all maturities are perfectly correlated.

SUMMARY OF KEY FINDINGS/RESULTS AND CONCLUSIONS

Bond prices depend on only one random variable, the spot interest rate, which serves as an instrument variable for underlying technological uncertainty. The bond price is a decreasing convex function of mean interest rate level and an increasing concave (decreasing convex) function of the speed of adjustment if the interest rate is greater (less) than the mean Current and future interest rates play a predominant role in determining term structure under special conditions.

The bond price is a decreasing convex function of the interest rate and an increasing (decreasing) function of time (maturity). Bond prices are an increasing concave function of the market risk parameter as higher values of this parameter indicate a greater covariance of interest rates with wealth. With large market risk it is more likely that bond prices will be higher when wealth is low and, hence, has greater marginal utility. Bond price is an increasing concave function of interest rate variance. Bonds are commonly quoted in terms of yields rather than prices. High long term interest rates reflect investors' fears of future inflation, recognizing that future monetary policy and economic conditions could be much different.

Tight monetary policy results in short term interest rates being higher than longer term rates. This occurs as a shortage of money and credit drives up the cost of short term capital. Long term rates stay lower, as investors see an eventual loosening of monetary policy and declining inflation. This increases the demand for long term bonds which lock in the higher long term rates.

CRITICAL EVALUATION OF THE PAPER

The framework within which CIR develop their continuous-time valuation model can briefly be described as follows: there are infinitely lived and identical individuals who maximize the discounted expected utility of consumption of a single good, which is produced stochastically from a finite number of technologies, each exhibiting constant stochastic returns to scale.

The individuals' wealth are totally invested in these firms, and they each choose a consumption rule and an investment allocation rule in maximizing their expected utility. The values of the firms in the economy evolve continuously as a vector Ito process, whose drift rate and covariance matrix depend on the evolution of a vector of state variables. The evolution of this vector of state variables is itself governed by a system of stochastic differential equations; therefore, the future investment opportunities in this model are stochastic.

The environment is competitive and frictionless; a riskless asset (which is in zero net supply) and the firms' shares are available for continuous trading with no transaction costs or taxes. The CIR model provides a reasonable characterization of the real returns on nominal bills at least for maturities of 12 months or less. The model performs reasonably well when confronted with data on short term treasury bills. The model is a single factor model which does not allow for more variability of long term nominal yields.

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